12:49
12:57
8
Give 20 minutes

Quiz 6, Linear

Name: Key

Assume G is an $n \times n$ matrix.

1. (4 points) If the equation $G\mathbf{x} = \mathbf{y}$ has more than one solution for some $\mathbf{y}$ in $\mathbb{R}^n$, can the columns of G span $\mathbb{R}^n$? Why or why not?

If $Gx = y$ has more than one solution, that means that $x \mapsto Gx$ is not 1-1. By the invertible matrix theorem G cannot be onto, so the columns of G do not span $\mathbb{R}^n$.

2. (2 points) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Prove that $\det(kA) = k^2(\det A)$ where k is any real number.

$$kA = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} \Rightarrow \det(kA) = (k^2ad - k^2bc)$$
$$= k^2(ad - bc)$$
$$= k^2\left(\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$$
$$= k^2 \det A.$$

3. (4 points) Calculate $\det A$ for $A = \begin{bmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{bmatrix}$.

$$\det A = 0 \cdots + 0 \cdot \det \cdots - 3 \begin{vmatrix} 4 & -7 & 3 & -5 \\ 0 & 2 & 0 & 0 \\ 5 & 5 & 2 & -3 \\ 0 & 9 & -1 & 2 \end{vmatrix} + 0 \cdot \det \cdots - 0 \cdot \det \cdots$$

$$= -3 \left(-0 \cdot \det \cdots + 2 \begin{vmatrix} 4 & 3 & -5 \\ 5 & 2 & -3 \\ 0 & -1 & 2 \end{vmatrix} - 0 \det [\cdots] + 0 \cdot \det [\cdots] \right)$$

$$= -6 \det \begin{vmatrix} 4 & 3 & -5 \\ 5 & 2 & -3 \\ 0 & -1 & 2 \end{vmatrix}$$

$$= -6 \cdot (-1)$$

$$= \boxed{6}$$

$$\det \begin{vmatrix} 4 & 3 & -5 \\ 5 & 2 & -3 \\ 0 & -1 & 2 \end{vmatrix} = \begin{matrix} 4 & 3 & -5 & 4 & 3 \\ 5 & 2 & -3 & 5 & 2 \\ 0 & -1 & 2 & 0 & -1 \end{matrix}$$

$$= 16 + (0) + 25 - 0 - 12 - 30$$

$$41 - 42 = -1$$